

**Highfield Church of England Primary School  
Calculation Progression**

<b>Representations and models to support addition and subtraction</b>		
<b>Name</b>	<b>Example</b>	<b>Benefits</b>
Part-Whole model		<p>This part-whole model supports children in their understanding of aggregation and partitioning. When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total. When the whole is complete and at least one of the parts is empty, children use partitioning to find the missing part. Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns. In KS2, children can apply this understanding to add and subtract fractions, decimals and percentages.</p>
Bar Model (single)  Concrete		<p>This is another type of part-whole model that can support children in representing calculations. Cubes and counters can be used in a line as a concrete representation of it.</p> <p>In KS2, children can use the bar model to represent larger numbers, decimals and fractions.</p>



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<p>Bar Model (single) Discrete</p>		<p>Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.</p>
<p>Bar Model (single) Combination</p>		<p>The combination bar model can support children to calculate by counting on from the larger number. It is a good step towards the continuous bar model.</p>
<p>Bar Model (single) Continuous</p>		<p>Continuous bar models are useful for a range of values. Each rectangle represents a number. Wherever there is a question mark, indicates the value that needs to be found.</p>



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<p>Bar Model (multiple) Discrete</p>		<p>The bar model can evolve from a single one and become the multiple bar model which is a good way to compare quantities.</p> <p>Two or more bars can be drawn, with a bracket labelling the whole. Smaller numbers can be represented with a discrete bar model. Continuous bar models are more effective when working with bigger numbers.</p>
<p>Bar Model (multiple) Continuous</p>		<p>Multiple bar models can also be used to represent the difference in subtraction with an arrow can be used to model the difference.</p> <p>Younger children can use cubes and a discrete model to find the difference. This also supports children to see how counting on can help when finding the difference.</p>



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Number shapes  
(numicon)

<b>1+9</b>	<b>2+8</b>	<b>3+7</b>	<b>4+6</b>

Number shapes can be useful to support children to subitise numbers. They can also be useful to explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together to make the whole. As children use them more often, they can start to subitise the total due to being familiar with the resource.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, they will be able to subitise due to the familiarity with the shapes.

They are also a useful resource to work systematically to find number bonds (ie numbers added together to make another number). As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.



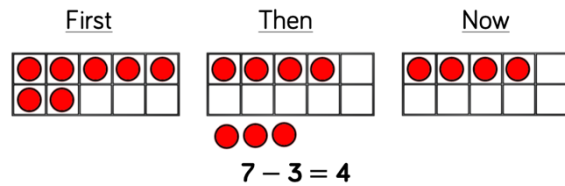
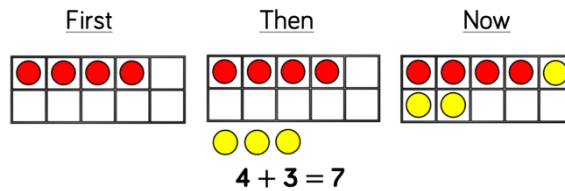
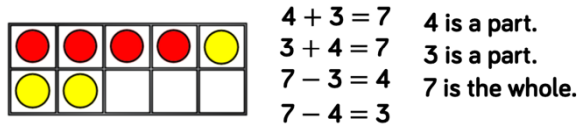
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<p>Cubes</p>	<p><math>7 = 4 + 3</math></p> <p><math>7 = 3 + 4</math></p> <p><math>7 - 3 = 4</math></p> <p><math>7 - 3 = 4</math></p> <p><math>7 - 3 = 4</math></p>	<p>Cubes can be useful to support children with addition and subtraction of one-digit numbers.</p> <p>With addition, children can use two different colours of cubes to represent the numbers before calculating the whole.</p> <p>When subtracting, children can start with the whole and then remove the number of cubes that they need to find the answer. This model of subtraction is reduction or take away.</p> <p>Cubes can also be useful to look at subtraction as difference. Here both numbers are made and then lined up to find the difference between the numbers.</p> <p>Cubes are predominately useful for smaller numbers.</p>
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Ten Frames  
(within 10)



When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

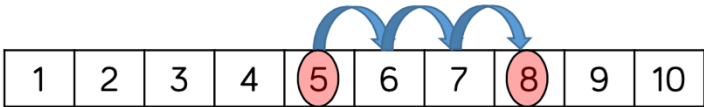
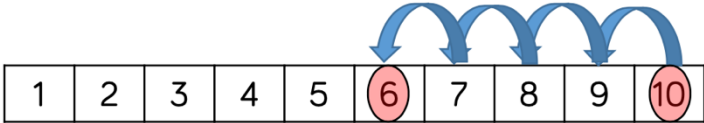
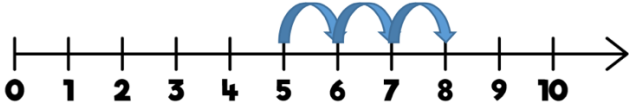
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning. Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number. Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage.

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<p>Ten Frames (within 20)</p>		<p>When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition. When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction. When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.</p>
<p>Bead Strings</p>		<p>Bead strings to 10 are very effective at helping children to investigate number bonds to 10. They can also help the children to do it systematically by moving one bead at a time.</p>



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		<p>Bead strings to 20 work in a similar way and they can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.</p> <p>Bead strings to 100 are grouped in tens and can support children in number bonds to 100. Bead strings can show a link to addition to the next 10 on number lines.</p>
Number Tracks	<p style="text-align: center;"><b><math>5 + 3 = 8</math></b></p>  <p style="text-align: center;"><b><math>10 - 4 = 6</math></b></p> 	<p>Number tracks are useful to support children in their understanding of augmentation and reduction.</p> <p>In adding on a number track, children can place a counter on the starting number and then count on to find the total. In subtracting, the children count back to find their answer.</p> <p>Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.</p>
Basic number lines (labelled)	<p style="text-align: center;"><b><math>5 + 3 = 8</math></b></p> 	<p>Labelled number lines support children in addition and subtraction. They can count on or back in ones, up or down the line. It directly links and builds on the use of the number track.</p>



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<p>Number lines (labelled)</p>	<p><math>8 + 7 = 15</math>  <math>2 \quad 5</math>  <math>+ 2 \quad + 5</math></p> <p><math>14 - 6 = 8</math>  <math>4 \quad 2</math>  <math>- 2 \quad - 4</math></p>	<p>Number lines can progress further by using the skill of partitioning. The children can jump to the nearest 10 and then jumping to the total. The smaller number is partitioned to support children to make a number bond to 10 and to then add on to the remaining part. Subtracting works in a similar way.</p>
<p>Number lines (blank)</p>	<p><math>35 + 37 = 72</math>  <math>+ 5 \quad + 30 \quad + 2</math>      35 40 70 72</p> <p><math>35 + 37 = 72</math>  <math>+ 5 \quad + 32</math>      35 40 72</p> <p><math>72 - 35 = 37</math>  <math>+ 5 \quad + 30 \quad + 2</math>      35 40 70 72</p>	<p>Blank number lines provide children with a structure to add and subtract numbers in smaller parts.</p> <p>They develop on from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the 10s and 1s separately.</p> <p>Blank number lines can also be used to subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up</p>

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		<p>the parts they have counted on to find the difference between the numbers.</p>
<p>Straws (for adding)</p>	<p><b><math>7 + 6 = 13</math></b></p> <p>bundle together groups of 10</p>	<p>Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.</p> <p>When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.</p>
<p>Straws (for subtracting)</p>	<p><b><math>42 - 17 = 25</math></b></p> <p>unbundle group of 10 straws</p>	<p>Straws can also aid in subtracting. When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones. Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.</p>



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<p>Base 10/ Dienes (addition)</p>		<p>Base 10/ Dienes are a fantastic aid to support the children's understanding of column addition.</p> <p><b>Important- children need to write out their calculations alongside using or drawing the dienes so they can see the clear links between the written method and the model.</b></p> <p>Children should first add without an exchange before moving on to addition with exchange. This is not as useful a representation with larger numbers simply due to the size of the dienes. Therefore, replace dienes with place value counters.</p>
<p>Base 10/ Dienes (subtraction)</p>		<p>Similarly, Base 10/ Dienes effectively supports understanding of column subtraction.</p> <p><b>Important- children need to write out their calculations alongside using or drawing the dienes so they can see the clear links between the written method and the model.</b></p> <p>Children should first subtract without an exchange before moving on to subtraction with exchange. When making the model, children should make the minuend using dienes, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough</p>



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		<p>ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently. This model is efficient with up to 4-digit numbers.</p> <p>When working with larger numbers, swap out dienes for place value counters.</p>																		
<p>Place Value Counters (addition)</p>	<div style="display: flex; align-items: center;"> <table border="1" style="margin-right: 20px;"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>100 100 100</td> <td>10 10 10 10 10 10 10 10</td> <td>1 1 1 1</td> </tr> <tr> <td>100 100</td> <td>10 10 10</td> <td>1 1 1 1 1 1 1</td> </tr> </tbody> </table> <div style="margin-right: 20px;"> <math display="block">\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 11 \end{array}</math> </div> </div> <div style="display: flex; align-items: center; margin-top: 20px;"> <table border="1" style="margin-right: 20px;"> <thead> <tr> <th>Ones</th> <th>Tenths</th> <th>Hundredths</th> </tr> </thead> <tbody> <tr> <td>1 1 1</td> <td>0.1 0.1 0.1 0.1 0.1 0.1</td> <td>0.01 0.01 0.01 0.01 0.01</td> </tr> <tr> <td>1 1</td> <td>0.1 0.1 0.1 0.1</td> <td>0.01</td> </tr> </tbody> </table> <div style="margin-right: 20px;"> <math display="block">\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}</math> </div> </div>	Hundreds	Tens	Ones	100 100 100	10 10 10 10 10 10 10 10	1 1 1 1	100 100	10 10 10	1 1 1 1 1 1 1	Ones	Tenths	Hundredths	1 1 1	0.1 0.1 0.1 0.1 0.1 0.1	0.01 0.01 0.01 0.01 0.01	1 1	0.1 0.1 0.1 0.1	0.01	<p>Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.</p> <p>Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.</p> <p>When adding money, children can also use coins to support their understanding. It is important that</p>
Hundreds	Tens	Ones																		
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100 100	10 10 10	1 1 1 1 1 1 1																		
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1 1	0.1 0.1 0.1 0.1	0.01																		



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		<p>children consider how the coins link to the written calculation especially when adding decimal amounts.</p>														
<p>Place Value Counters (Subtraction)</p>	<div style="display: flex; flex-direction: column; align-items: center;"> <table border="1" style="margin-bottom: 20px;"> <thead> <tr> <th style="width: 33%;">Hundreds</th> <th style="width: 33%;">Tens</th> <th style="width: 33%;">Ones</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;"> </td> <td style="text-align: center;"> </td> <td style="text-align: center;"> </td> </tr> </tbody> </table> <div style="margin-bottom: 20px;"> <math display="block">\begin{array}{r} 652 \\ - 207 \\ \hline 445 \end{array}</math> </div> <table border="1" style="margin-bottom: 20px;"> <thead> <tr> <th style="width: 25%;">Thousands</th> <th style="width: 25%;">Hundreds</th> <th style="width: 25%;">Tens</th> <th style="width: 25%;">Ones</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;"> </td> <td style="text-align: center;"> </td> <td style="text-align: center;"> </td> <td style="text-align: center;"> </td> </tr> </tbody> </table> <div> <math display="block">\begin{array}{r} 4357 \\ - 2735 \\ \hline 1622 \end{array}</math> </div> </div>	Hundreds	Tens	Ones				Thousands	Hundreds	Tens	Ones					<p>Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.</p> <p>Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.</p> <p>When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.</p>
Hundreds	Tens	Ones														
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<b>Statutory guidance – Written Methods for addition</b>																										
$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ \hline 1 \end{array}$ $\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ \hline 1 \end{array}$	<p>From Year 3, we encourage children to use the formal column method when calculating alongside straws, base 10/ dienes or place value counters.</p>																									
<table border="1"><tbody><tr><td></td><td>1</td><td>3</td><td>7</td><td>8</td></tr><tr><td>+</td><td>2</td><td>1</td><td>4</td><td>8</td></tr><tr><td colspan="5"><hr/></td></tr><tr><td></td><td>3</td><td>5</td><td>2</td><td>6</td></tr><tr><td></td><td>1</td><td>1</td><td></td><td></td></tr></tbody></table>		1	3	7	8	+	2	1	4	8	<hr/>						3	5	2	6		1	1			<p>In Year 4, we ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.</p>
	1	3	7	8																						
+	2	1	4	8																						
<hr/>																										
	3	5	2	6																						
	1	1																								



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<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>1</td><td>0</td><td>4</td><td>3</td><td>2</td><td>8</td></tr> <tr><td>+</td><td>6</td><td>1</td><td>7</td><td>3</td><td>1</td></tr> <tr><td colspan="6"><hr/></td></tr> <tr><td>1</td><td>6</td><td>6</td><td>0</td><td>5</td><td>9</td></tr> </table> <p style="text-align: center;">1</p>	1	0	4	3	2	8	+	6	1	7	3	1	<hr/>						1	6	6	0	5	9	<p>When our children get to Year 5/6, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.</p>
1	0	4	3	2	8																				
+	6	1	7	3	1																				
<hr/>																									
1	6	6	0	5	9																				
<p><b>Statutory guidance – Written Methods for subtraction</b></p>																									
$  \begin{array}{r}  \overset{5}{\cancel{6}}\overset{1}{5} \\  - 28 \\  \hline  37  \end{array}  $	<p>From Year 3, we encourage children to use the formal column method when calculating alongside straws, base 10 / dienes or place value counters.</p>																								
$  \begin{array}{r}  \overset{3}{\cancel{4}}\overset{1}{3}5 \\  - 273 \\  \hline  162  \end{array}  $	<p>In Year 4, we ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.</p>																								



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	2	9	<del>3</del> <sup>4</sup>	<del>1</del> <sup>3</sup>	8	2	When our children get to Year 5/6, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.
-	1	8	2	5	0	1	
	1	1	1	8	8	1	



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**Below is an overview of which year group uses which image for addition**

<b>Skill</b>	<b>Year</b>	<b>Representation and model</b>
Add two 1-digit numbers to 10	EYFS / 1	<ol style="list-style-type: none"> <li>1. Part-whole model</li> <li>2. Bar model</li> <li>3. Number shapes</li> <li>4. Ten frames (within 10)</li> <li>5. Bead strings (10)</li> <li>6. Number tracks</li> </ol>
Add 1 and 2-digit numbers to 20	EYFS / 1	<ol style="list-style-type: none"> <li>1. Part-whole model</li> <li>2. Bar model</li> <li>3. Number shapes</li> <li>4. Tens frames (within 20)</li> <li>5. Bead strings (20)</li> <li>6. Number tracks</li> <li>7. Number lines (labelled)</li> <li>8. Straws</li> </ol>
Add three 1-digit numbers	2	<ol style="list-style-type: none"> <li>1. Part-whole model</li> <li>2. Bar model</li> <li>3. Tens frames (within 20)</li> <li>4. Number shapes</li> </ol>
Add 1 and 2-digit numbers to 100	2	<ol style="list-style-type: none"> <li>1. Part-whole model</li> <li>2. Bar model</li> <li>3. Number lines (labelled)</li> <li>4. Number lines (blank)</li> </ol>



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		5. Straws 6. Hundred square
Add two 2-digit numbers	2	1. Part-whole model 2. Bar model 3. Number lines (blank) 4. Straws 5. Base 10 / Dienes 6. Place value counters
Add with up to 3-digits	3	1. Part-whole model 2. Bar model 3. Base 10 / Dienes 4. Place value counters 5. Column addition
Add with up to 4-digits	4	1. Part-whole model 2. Bar model 3. Base 10 / Dienes 4. Place Value counters 5. Column addition
Add with more than 4 digits	5/6	1. Part-whole model 2. Bar model 3. Place value counters 4. Column addition
Add with up to 3 decimal places	5	1. Part-whole model 2. Bar model



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- |  |  |   |
|--|--|---|
|  |  | <ul style="list-style-type: none"> <li>3. Place value counters</li> <li>4. Column addition</li> </ul> |
|--|--|---|

**Below is an overview of which year group uses which image for subtraction**

<b>Skill</b>	<b>Year</b>	<b>Representations and models</b>
Subtract two 1-digit numbers to 10	EYFS/ 1	<ul style="list-style-type: none"> <li>1. Part-whole model</li> <li>2. Bar model</li> <li>3. Number shapes</li> <li>4. Ten frames (within 10)</li> <li>5. Bead strings (10)</li> <li>6. Number tracks</li> </ul>
Subtract 1 and 2 digit numbers to 20	EYFS / 1	<ul style="list-style-type: none"> <li>1. Part-whole model</li> <li>2. Bar model</li> <li>3. Number shapes</li> <li>4. Ten frames (within 20)</li> <li>5. Bead string (20)</li> <li>6. Number tracks</li> <li>7. Number lines (labelled)</li> <li>8. Straws</li> </ul>
Subtract 1 and 2 digit numbers to 100	2	<ul style="list-style-type: none"> <li>1. Part-whole model</li> <li>2. Bar model</li> <li>3. Number lines (labelled)</li> </ul>



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		4. Number lines (blank) 5. Straws 6. Hundred square
Subtract two 2-digit numbers	2	1. Part-whole model 2. Bar model 3. Number lines (blank) 4. Straws 5. Base 10 / dienes 6. Place value counters
Subtract with up to 3-digits	3	1. Part-whole model 2. Bar model 3. Base 10 / dienes 4. Place value counters 5. Column subtraction
Subtract with up to 4-digits	4	1. Part-whole model 2. Bar model 3. Base 10 / dienes 4. Place value counters 5. Column subtraction
Subtract with more than 4 digits	5	1. Part-whole model 2. Bar model 3. Place value counters 4. Column subtraction
Subtract with up to 3 decimal places	5	1. Part-whole model



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		2. Bar model 3. Place value counters 4. Column subtraction
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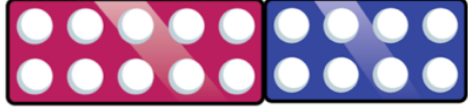
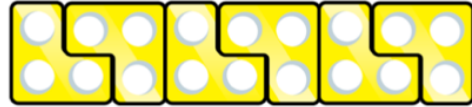
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**Representations and models to support multiplication and division**

Name	Example	Benefits
Single Bar Model (for multiplication)		Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.
Single Bar Model (for division)		Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.
Multiple Bar Model		Occasionally, children may look at scaling problems. When this happens, more than one bar model is useful to represent the problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there? The multiple bar model provides an opportunity to compare the groups.



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<p>Number shapes (for multiplication)</p>	 $5 \times 4 = 20$ $4 \times 5 = 20$  $5 \times 4 = 20$ $4 \times 5 = 20$	<p>The use of number shapes (numicon) helps to establish the children's understanding of multiplication as repeated addition.</p> <p>The children can built multiplications in a row using the number shapes.</p>
<p>Number shapes (for division)</p>	 $18 \div 3 = 6$ 	<p>When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.</p>

### Highfield Church of England Primary School Calculation Progression

Bead Strings



$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

$$15 \div 3 = 5$$



$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

$$15 \div 5 = 3$$



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

Bead strings also help the understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more.

**Important- Encourage the children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.**

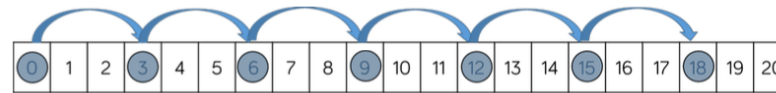
Children can also use the bead string to count forwards and backwards in multiples whilst moving the beads.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4, make 20 and then group the beads into groups of four. The children then count the number of groups they have to find the answer.



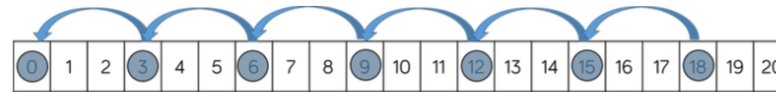
### Highfield Church of England Primary School Calculation Progression

#### Number Tracks



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

Number tracks are helpful to support children in counting in multiples both forwards and backwards. Using a counting aid like a translucent counter or cube can support the children to keep track of their counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

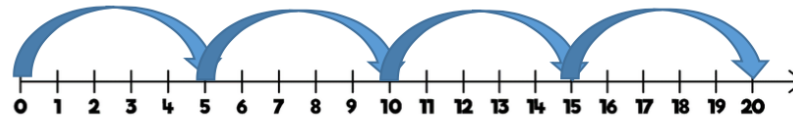
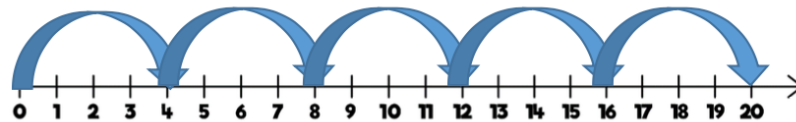
When dividing, children place their counter on the number they are dividing and then count back in jumps of the number they are dividing until they reach 0. Children then record how many jumps they have made to find the answer to the division.

**Important- Number tracks are only useful for smaller multiples.**



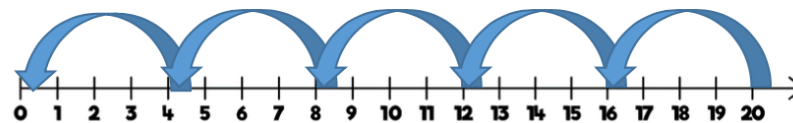
### Highfield Church of England Primary School Calculation Progression

Number Lines (labelled)



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

Labelled number lines are a supportive aid for children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

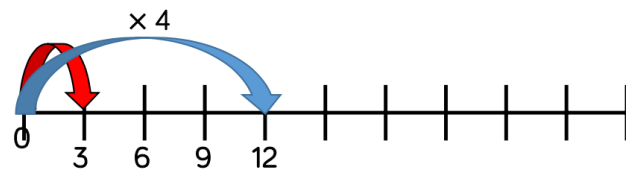
When dividing, start at the number they are dividing and then count back in jumps of the number they are dividing by until they reach 0. The children then record how many jumps they have made to find the answer to the division.

**Important- Labelled number lines are only useful for smaller multiples.**

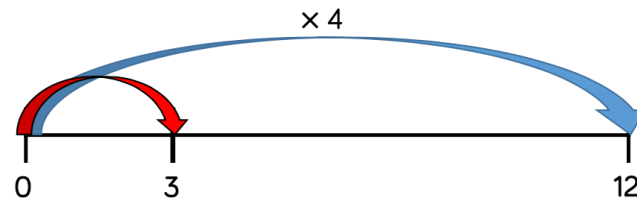


### Highfield Church of England Primary School Calculation Progression

Number Lines (blank)



A red car travels 3 miles.  
A blue car 4 times further.  
How far does the blue car travel?



A blue car travels 12 miles.  
A red car 4 times less.  
How far does the red car travel?

Children can use blank number lines to represent scaling as multiplication or division.

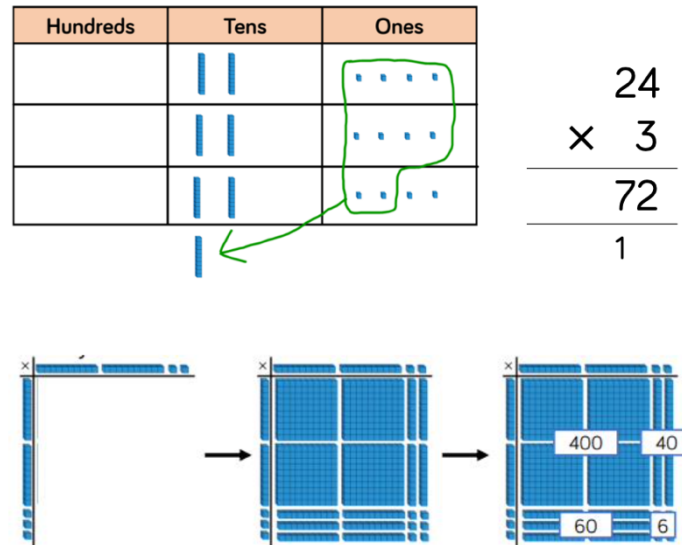
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number links without intervals can also be used.



### Highfield Church of England Primary School Calculation Progression

Base 10 / Dienes (for multiplication)



Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication.

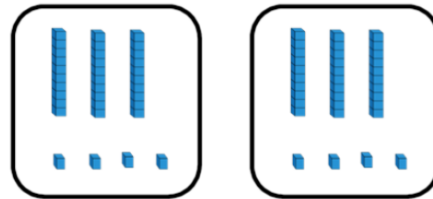
**Important- Children should write out their calculation alongside the equipment so they can see how they concrete and written representations match.**

It also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces. This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

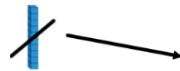


### Highfield Church of England Primary School Calculation Progression

Base 10 / Dienes (for division)

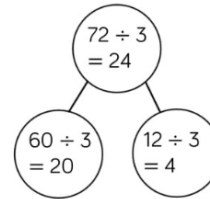


$$68 \div 2 = 34$$



Tens	Ones

$$72 \div 3 = 24$$



Using Base 10 / Dienes is an effective way to support children in their understanding of division.

Important- when numbers become larger, move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share them between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, children could record the part-whole model so they can consider how the number has been partitioned in order to divide.



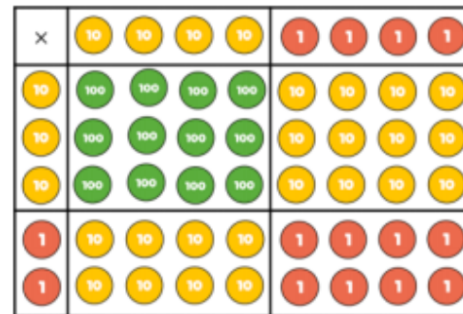
**Highfield Church of England Primary School  
Calculation Progression**

Place value counters (for multiplication)



$$\begin{array}{r} 34 \\ \times 5 \\ \hline 170 \\ \hline 12 \end{array}$$

Place value counters develop children's understanding of column multiplication. It is important they write their calculation alongside the equipment so they can see how the concrete resource and the written match.



$$\begin{array}{r} 44 \\ \times 32 \\ \hline 88 \\ 800 \\ \hline 1408 \\ \hline 1 \end{array}$$

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

### Highfield Church of England Primary School Calculation Progression

Place value counters (for division)

They are also an effective way to support children's understanding of division.

When working with smaller numbers, the children can use them to share between groups. They start by sharing the larger place value column and work from left to right. If any counters are left, they exchange the counter e.g. exchange one ten for ten ones. It is worth noting that this method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters



**Highfield Church of England Primary School  
Calculation Progression**

left over after grouping, they exchange the counter e.g. exchange one hundred for ten tens.

**Statutory Guidance for Multiplication**

	H	T	O		
		3	4		
x			5		
		2	0	(5 × 4)	
+	1	5	0	(5 × 30)	
	1	7	0		

	H	T	O	
		3	4	
x			5	
	1	7	0	
	1	2		

In Year 3, informal methods and the expanded method are used before moving on to the short multiplication method in Year 4.

	H	T	O
	2	4	5
x			4
	9	8	0
	1	2	

In Year 4, we encourage children to move towards the short, formal written method.



**Highfield Church of England Primary School**  
**Calculation Progression**

<table border="1"> <thead> <tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th></tr> </thead> <tbody> <tr><td></td><td>1</td><td>8</td><td>2</td><td>6</td></tr> <tr><td>x</td><td></td><td></td><td></td><td>3</td></tr> <tr><td></td><td>5</td><td>4</td><td>7</td><td>8</td></tr> </tbody> </table> <p style="text-align: center;">2                      1</p>		Th	H	T	O		1	8	2	6	x				3		5	4	7	8	<table border="1"> <thead> <tr><th></th><th>H</th><th>T</th><th>O</th></tr> </thead> <tbody> <tr><td></td><td></td><td>2</td><td>2</td></tr> <tr><td>x</td><td></td><td>3</td><td>1</td></tr> <tr><td></td><td></td><td>2</td><td>2</td></tr> <tr><td></td><td>6</td><td>6</td><td>0</td></tr> <tr><td></td><td>6</td><td>8</td><td>2</td></tr> </tbody> </table>		H	T	O			2	2	x		3	1			2	2		6	6	0		6	8	2	<p>In Year 5, children can use the grid method as an initial written method before moving on to the formal written multiplication method.</p>										
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x	20	2																																																						
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**Highfield Church of England Primary School**  
**Calculation Progression**

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		1	3																												
	4	5	1	2																											
<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td>0</td> <td>3</td> <td>6</td> </tr> <tr> <td>1</td> <td>2</td> <td style="border-left: 2px solid black;">4</td> <td style="border-left: 2px solid black;">3</td> <td>2</td> </tr> <tr> <td></td> <td>-</td> <td>3</td> <td>6</td> <td>0</td> </tr> <tr> <td></td> <td></td> <td></td> <td>7</td> <td>2</td> </tr> <tr> <td></td> <td>-</td> <td></td> <td>7</td> <td>2</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>0</td> </tr> </table> <div style="margin-left: 20px;"> <p>(x30)</p> <ul style="list-style-type: none"> <li>12 × 1 = 12</li> <li>12 × 2 = 24</li> <li>12 × 3 = 36</li> <li>12 × 4 = 48</li> <li>12 × 5 = 60</li> </ul> <p>(x6)</p> <ul style="list-style-type: none"> <li>12 × 6 = 72</li> <li>12 × 7 = 84</li> <li>12 × 8 = 96</li> <li>12 × 7 = 108</li> <li>12 × 10 = 120</li> </ul> </div>			0	3	6	1	2	4	3	2		-	3	6	0				7	2		-		7	2					0	<p>Children can also divide by 2-digit numbers using long division. Children can write out multiples to support their calculations with larger remainders.</p>
		0	3	6																											
1	2	4	3	2																											
	-	3	6	0																											
			7	2																											
	-		7	2																											
				0																											



**Highfield Church of England Primary School**  
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			2	4	$\frac{4}{5}$
1	5	3	7	2	
	-	3	0	0	
			7	2	
	-		6	0	
			1	2	

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.



**Highfield Church of England Primary School  
Calculation Progression**

**Below is an overview of which year group uses which image for multiplication**

<b><u>Skill</u></b>	<b><u>Year</u></b>	<b><u>Representations and models</u></b>
<u>Solve one-step problems with multiplication</u>	<u>1 / 2</u>	1. <u>Bar model</u> 2. <u>Number shapes</u> 3. <u>Counters</u> 4. <u>Ten frames</u> 5. <u>Bead strings</u> 6. <u>Number lines</u>
<u>Multiply 2-digit by 1-digit numbers</u>	<u>3 / 4</u>	1. <u>Place value counters</u> 2. <u>Base 10 / Dienes</u> 3. <u>Expanded written method</u> 4. <u>Short written method</u>
<u>Multiply 3-digit by 1-digit numbers</u>	<u>4</u>	1. <u>Place value counters</u> 2. <u>Base 10 / Dienes</u> 3. <u>Short written method</u>
<u>Multiply 4-digit by 1-digit numbers</u>	<u>5</u>	1. <u>Place value counters</u> 2. <u>Short written method</u>
<u>Multiply 2-digit by 2-digit numbers</u>	<u>5</u>	1. <u>Place value counters</u> 2. <u>Base 10 / Dienes</u> 3. <u>Short written method</u> 4. <u>Grid method</u>
<u>Multiply 2-digit by 3-digit numbers</u>	<u>5</u>	1. <u>Place value counters</u> 2. <u>Short written method</u> 3. <u>Grid method</u>



**Highfield Church of England Primary School**  
**Calculation Progression**

Multiply 2-digit by 4-digit numbers	$5 / 6$	1. Formal written method
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**Below is an overview of which year group uses which image for division**

<b>Skill</b>	<b>Year</b>	<b>Representations and models</b>
Solve one-step problems with division (sharing)	1 / 2	<ol style="list-style-type: none"> <li>1. Bar model</li> <li>2. Real life objects</li> <li>3. Arrays</li> <li>4. Counters</li> </ol>
Solve one-step problems with division (grouping)	1 / 2	<ol style="list-style-type: none"> <li>1. Real life objects</li> <li>2. Number shapes</li> <li>3. Bead strings</li> <li>4. Ten frames</li> <li>5. Number lines</li> <li>6. Arrays</li> <li>7. Counters</li> </ol>
Divide 2-digits by 1-digit (no exchange sharing)	3	<ol style="list-style-type: none"> <li>1. Straws</li> <li>2. Base 10 / Dienes</li> <li>3. Bar model</li> <li>4. Place value counters</li> <li>5. Part-whole model</li> </ol>
Divide 2-digits by 1-digit (sharing with exchange)	3	<ol style="list-style-type: none"> <li>1. Straws</li> <li>2. Base 10 / Dienes</li> <li>3. Bar model</li> <li>4. Place value counters</li> </ol>



**Highfield Church of England Primary School**  
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		5. Part-whole model
Divide 2-digits by 1-digit (sharing with numbers)	3 / 4	1. Straws 2. Base 10 / Dienes 3. Bar model 4. Place value counters 5. Part-whole model
Divide 2-digits by 1-digit (grouping)	4 / 5	1. Base 10 / Dienes 2. Bar model 3. Place value counters 4. Written short division
Divide 3-digits by 1-digit (sharing with exchange)	4	1. Base 10 / Dienes 2. Bar model 3. Place value counters 4. Part-whole model
Divide 3-digits by 1-digit (grouping)	4 / 5	1. Place value counters 2. Counters 3. Place value grid 4. Written short division
Divide 4-digits by 1-digit (grouping)	5	1. Place value counters 2. Counters 3. Place value grid 4. Written short division
Divide multi-digits by 2-digits (short division)	6	1. Written short division 2. List of multiples



**Highfield Church of England Primary School**  
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Divide multi-digits by 2-digits (long division)	6	1. Written long division 2. List of multiples
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**Highfield Church of England Primary School**  
**Calculation Progression**

## Glossary of terms used in addition and subtraction section

Addend	A number to be added to another.	Partitioning	Splitting a number into its component parts.
Aggregation	Combining two or more quantities or measures to find a total.	Reduction	Subtraction as take away.
Augmentation	Increasing a quantity or measure by another quantity.	Subitise	Instantly recognise the number of objects in a small group without needing to count.
Commutative	Numbers can be added in any order.	Subtrahend	A number to be subtracted from another.
Complement	In addition, a number and its complement make a total (e.g. 30 is the complement to 70 to make 100).	Sum	The result of an addition.
Difference	The numerical difference between two numbers is found by comparing the quantity in each group.	Total	The aggregate or the sum found by addition.
Exchange	Change a number or expression for another of an equal value.		
Minuend	A quantity or number from which another is subtracted.		



**Highfield Church of England Primary School**  
**Calculation Progression**

## Glossary of terms used in multiplication and division section

Array	An ordered collection of counters, cubes or other item in rows and columns.	Product	The result of multiplying one number by another.
Commutative	Numbers can be multiplied in any order.	Quotient	The result of a division.
Dividend	In division, the number that is divided.	Remainder	The amount left over after a division when the divisor is not a factor of the dividend.
Divisor	In division, the number by which another is divided.	Scaling	Enlarging or reducing a number by a given amount, called the scale factor.
Exchange	Change a number or expression for another of an equal value.		
Factor	A number that multiplies with another to make a product.		
Multiplicand	In multiplication, a number to be multiplied by another.		
Partitioning	Splitting a number into its component parts.		